Econ 802
Answers to Final Exam

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1\. (a) Suppose both y^1 and y^2 maximize profit at the prices p, with $y^1 \neq y^2$. We can write $\pi^{max} = py^1 = py^2$. Now consider a convex combination $y^3 = ty^1 + (1-t)y^2$ where $0 < t < 1$. Since Y is convex $y^3 \in Y$. Also,
$py^3 = p[ty^1 + (1-t)y^2] = t\pi^{max} + (1-t)\pi^{max} = \pi^{max}$,
so y^3 also maximizes profit. But because Y is strictly convex, y^3 is in the interior of Y for $0 < t < 1$. This means we can find some nearby vector that is also feasible and has more of some outputs with the same input levels. Since this would increase profit, y^1, y^2, and y^3 cannot be profit maximizing. This contradiction shows that we cannot have $y^1 \neq y^2$ both maximizing profit, which implies the solution is unique.

(b) Let p^0 and p' be two different price vectors and write $y^0 = y(p^0)$, $y' = y(p')$. Define $p'' = tp^0 + (1-t)p'$ where $0 \leq t \leq 1$. Then we have

$\pi(p'') = p''y''$

where $y'' = y(p'')$ is the solution for the price vector p''. Thus $\pi(p'') = tp^0y'' + (1-t)p'y'' \leq t\pi(p^0) + (1-t)\pi(p')$ because $p^0y'' \leq p^0y^0 = \pi(p^0)$ and $p'y'' \leq p'y' = \pi(p')$. (follows because y^0 is optimal at p^0 and y' is optimal at p'). This shows that $\pi(p)$ is a convex function.

(c) From profit maximization it must be true that $p^1y^1 \geq p^1y^2$ and $p^2y^2 \geq p^2y^1$. Rewrite the first inequality as $-p^1y^2 \geq -p^1y^1$. Now sum the inequalities to get $p^2y^2 - p^1y^2 \geq p^2y^1 - p^1y^1$. This gives $\Delta p\, y^2 \geq \Delta p\, y^1 \Rightarrow \Delta p\,(y^2 - y^1) \geq 0$ or $\Delta p\, \Delta y \geq 0$. Thus if y_1 is an output ($y_1 > 0$), and $\Delta p_1 > 0$, $\Delta p_2 = 0$, we must have $\Delta y_1 \geq 0$ so supply curves cannot slope down. If y_2 is an input ($y_2 < 0$), and $\Delta p_1 = 0$, $\Delta p_2 > 0$, we must have $\Delta y_2 \geq 0$, which means that less of the input is used in absolute value terms. Thus input demand curves cannot slope up. These results all follow from the weak axiom of profit maximization (WAPM).

This is the case with two goods, but the result generalizes to any number of goods.

2. (a) Write $c(y)$ in place of $c(w,y)$ because the input prices w are not changing. Short run average cost is $\frac{c(y)}{y}$ and the slope of SRAC with respect to output is

$$\frac{d}{dy}\left[\frac{c(y)}{y}\right] = \frac{c'(y)}{y} - \frac{c(y)}{y^2}$$
$$= \frac{1}{y}\left[MC(y) - AC(y)\right]$$

So when $AC(y)$ is decreasing in y we must have $MC(y) < AC(y)$ and when $AC(y)$ is increasing in y we must have $MC(y) > AC(y)$.

(b) The output obtained from the fixed inputs is $a_f x_f$ where a_f is the vector of a_i coefficients associated with these inputs. We therefore need to produce $y - a_f x_f$ units of output using the variable inputs.

Set up the Lagrangean as follows:

$$L = -\sum_i w_i x_i + \lambda\left[\sum_i a_i x_i - (y - a_f x_f)\right] + \sum_i \mu_i x_i$$

Note that the minus sign in front reflects the fact that we are minimizing $\sum_i w_i x_i$, which is the same as maximizing $-\sum_i w_i x_i$. All summations are taken over the set of variable inputs only (the fixed inputs cannot be adjusted in the short run).

The first order conditions are

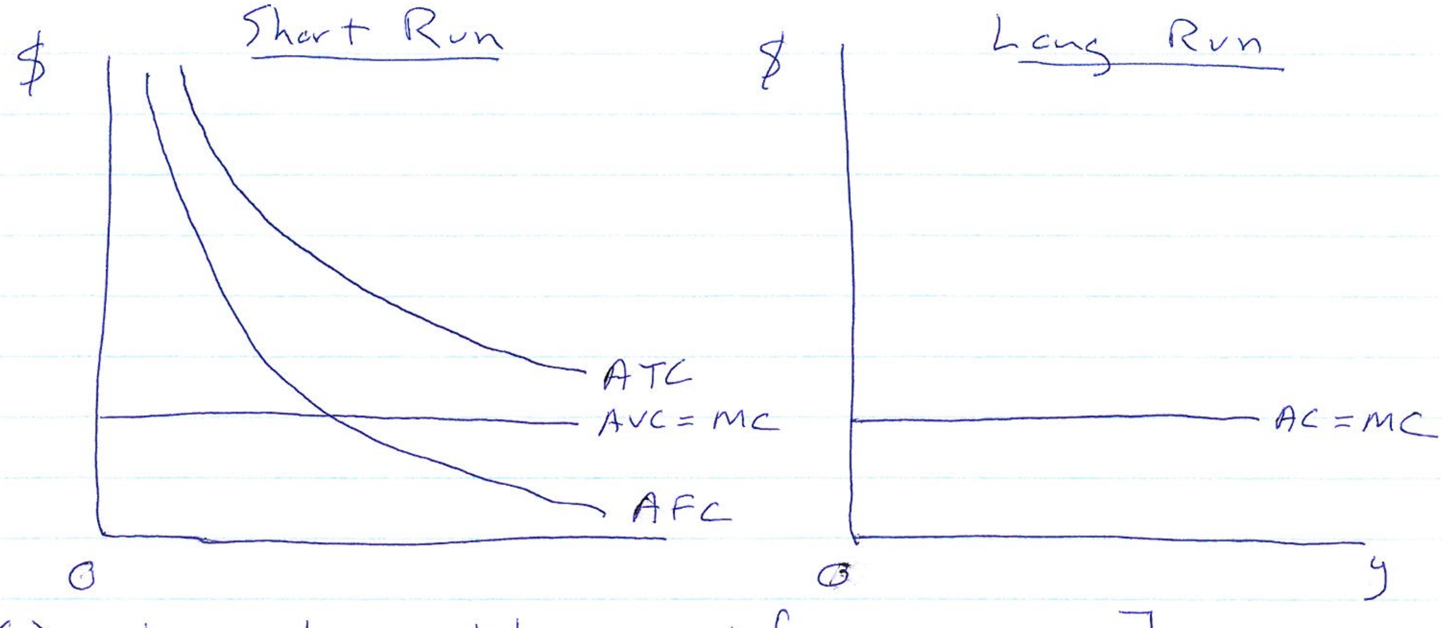
$$\frac{\partial L}{\partial x_i} = -w_i + \lambda a_i + \mu_i = 0 \quad \text{where } \mu_i \geq 0,\ x_i \geq 0, \text{ and } \mu_i x_i = 0.$$

If $x_i > 0$ so i is used at a positive level, we must have $\mu_i = 0$ and thus $\lambda = \frac{w_i}{a_i}$

If $x_i = 0$, we have $\mu_i \geq 0$ and thus $\lambda \leq \frac{w_i}{a_i}$

So the only variable inputs that can be used at a positive level are those with the minimum ratio $\frac{w_i}{a_i}$. Any input with a ratio $\frac{w_i}{a_i}$ higher than the minimum must be set equal to zero (note that λ does not have an i subscript).

(c) Linear production functions have constant returns to scale and therefore have flat average and marginal cost curves. This is true for the variable inputs in the short run, and it is also true when all inputs can be varied in the long run.



3. (a) $L = a \ln x_1 + b \ln x_2 - \lambda [p_1 x_1 + p_2 x_2 - m]$

FOC $\Rightarrow$ $\frac{a}{x_1} = \lambda p_1$ and $\frac{b}{x_2} = \lambda p_2$

Divide one equation by the other to get

$\frac{x_2}{x_1} = \frac{b}{a}\frac{p_1}{p_2}$. Substitute into budget constraint to get

$$x_1(p,m) = \left(\frac{a}{a+b}\right)\frac{m}{p_1} \quad \text{and}$$

$$x_2(p,m) = \left(\frac{b}{a+b}\right)\frac{m}{p_2}$$

So $v(p,m) = a \ln\left[\left(\frac{a}{a+b}\right)\left(\frac{m}{p_1}\right)\right] + b \ln\left[\left(\frac{b}{a+b}\right)\left(\frac{m}{p_2}\right)\right]$

(b) $L = -p_1 x_1 - p_2 x_2 + \lambda [a \ln x_1 + b \ln x_2 - u]$

FOC $\Rightarrow$ $\lambda a = p_1 x_1$ and $\lambda b = p_2 x_2$

Divide one equation by the other to get

$\frac{x_2}{x_1} = \frac{b}{a}\frac{p_1}{p_2}$. Substitute into utility constraint to get

$$h_1(p,u) = e^{\frac{u}{a+b}}\left[\frac{a}{b}\frac{p_2}{p_1}\right]^{\frac{b}{a+b}}$$

$$h_2(p,u) = e^{\frac{u}{a+b}}\left[\frac{b}{a}\frac{p_1}{p_2}\right]^{\frac{a}{a+b}}$$

So $e(p,u) = p_1 h_1(p,u) + p_2 h_2(p,u)$

$$= e^{\frac{u}{a+b}}\left[p_1\left(\frac{a}{b}\frac{p_2}{p_1}\right)^{\frac{b}{a+b}} + p_2\left(\frac{b}{a}\frac{p_1}{p_2}\right)^{\frac{a}{a+b}}\right]$$

(c) This has no effect on the Marshallian demands because it is just an increasing monotonic transformation of the utility function, and the new utility function represents the same preferences as the old one. However, it does decrease the Hicksian demands h_1 and h_2 if the utility level u in the constraint is left unchanged, due to the $e^{\frac{u}{a+b}}$ terms. If we also multiply the utility level u by the same positive constant, to reflect the use of a new utility function then there is no effect on the Hicksian demands either.

4. (a) We want to max $y_i + \ln x_i$ subject to $px_i = w(T - y_i)$. Use the constraint to solve for $y_i = T - \frac{p}{w} x_i$. Substitute into the utility function and max

$$T - \frac{p}{w} x_i + \ln x_i$$

with respect to x_i. This gives

FOC: $-\frac{p}{w} + \frac{1}{x_i} = 0$ or $x_i(p, w) = \frac{w}{p}$

Since there are n identical consumers, the market demand for widgets is just

$$X(p, w) = \sum_{i=1}^{n} x_i(p, w) = \frac{nw}{p}.$$

(b) Firm j maxes profit $pz_j - wF - wz_j^2$ if it enters the industry and produces $z_j > 0$.

FOC $\Rightarrow$ $p - 2wz_j = 0$ or $z_j(p, w) = \frac{p}{2w}$.

BUT we have to make sure that the resulting profit is non-negative because the firm has the option of not entering the industry and getting zero profit. Thus we need

$$p\left(\frac{p}{2w}\right) - wF - w\left(\frac{p}{2w}\right)^2 \geq 0$$

$$\text{or} \quad \frac{p^2}{2w^2} - \frac{p^2}{4w^2} \geq F \quad \text{or} \quad \frac{p^2}{4w^2} \geq F$$

$$\Rightarrow \quad p \geq 2w\sqrt{F}$$

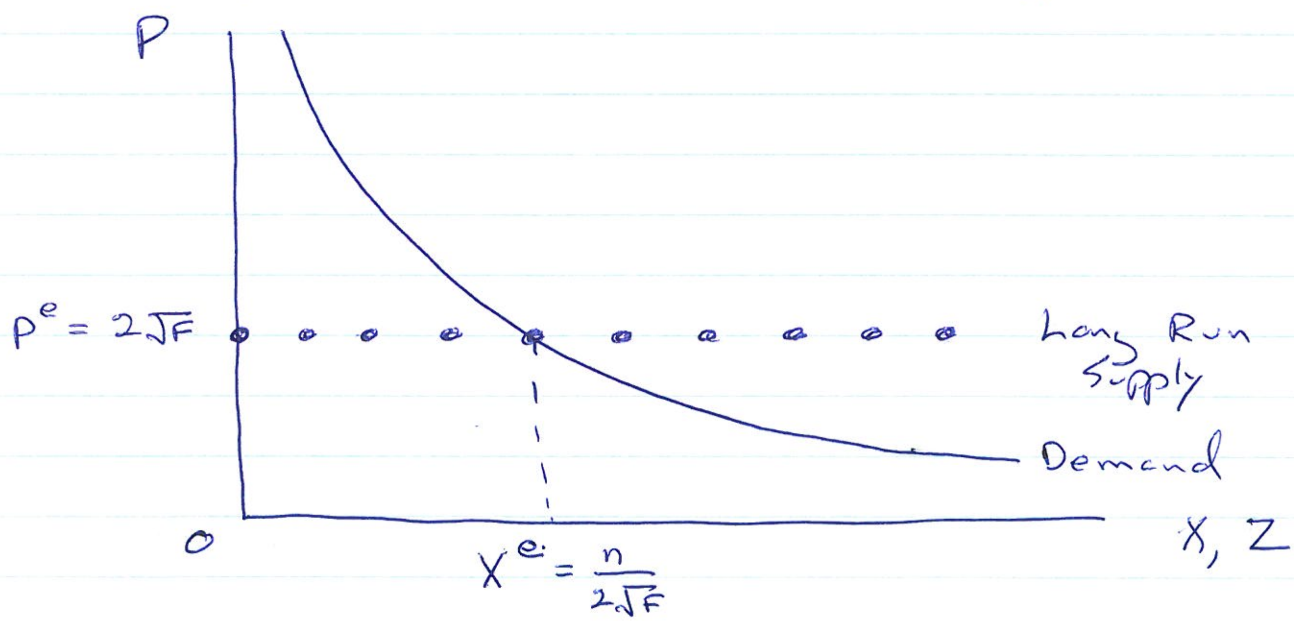
So the complete supply function is

$z_j(p,w) = 0$ if $p < 2w\sqrt{F}$ (firm stays out)

$z_j(p,w) = 0$ or $\frac{p}{2w}$ if $p = 2w\sqrt{F}$ (firm is indifferent)

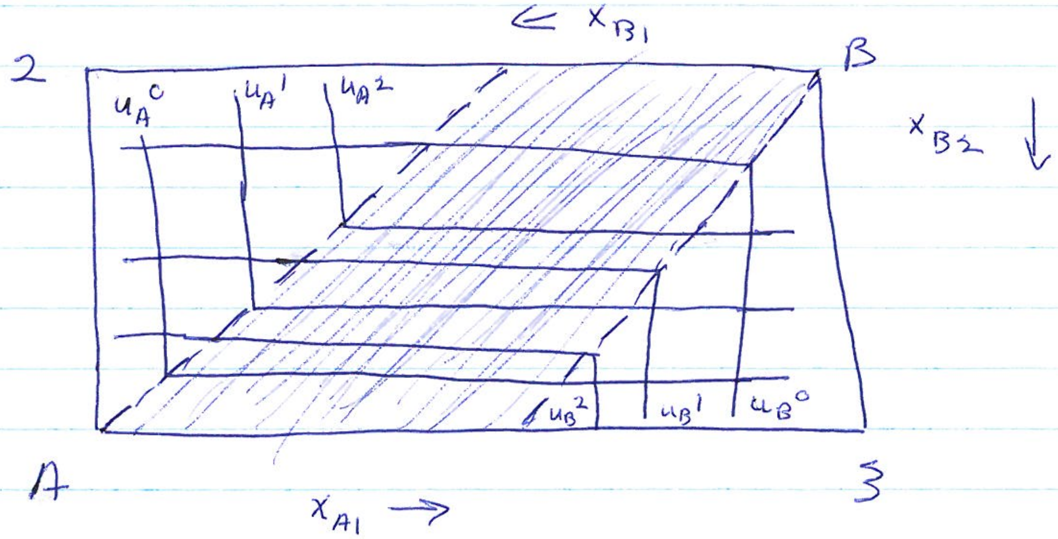
$z_j(p,w) = \frac{p}{2w}$ if $p > 2w\sqrt{F}$

(c) With $w = 1$, market demand is $X = \frac{n}{p}$. Firms stay out when $p < 2\sqrt{F}$. If the firms are price takers, they always enter when $p > 2\sqrt{F}$ because they can get positive profit. So the only possible equilibrium price is $p^e = 2\sqrt{F}$. The quantity demanded is $X = \frac{n}{2\sqrt{F}}$



When m firms are operating and the price is p^e, total supply is $Z = \frac{mp^e}{2} = m\sqrt{F}$.
Setting supply equal to demand gives
$$X = Z \quad \text{or} \quad m\sqrt{F} = \frac{n}{2\sqrt{F}} \Rightarrow m = \frac{n}{2F}.$$
Note that we are assuming this number of firms gives exactly zero profit (m is an integer)

5. (a)



Each person has Leontief utility, with the corner points for A along the 45° line from A's origin, and the corner points for B along the 45° line from B's origin. The set of Pareto efficient points is the shaded area between the two 45° lines (including those boundaries). At any point outside this region, it is possible to make both people better off simultaneously by moving diagonally toward the shaded area. For any point <u>in</u> the shaded area, it is impossible to make one person better off without making the other person worse off.

(b) Let's find the Marshallian demands for person A. We max $\min\{x_{A1}, x_{A2}\}$ subject to $p_1 x_1 + p_2 x_2 = m_A$. When the prices are both positive, we must be at a corner of the indifference curve where $x_{A1} = x_{A2}$. Substituting into the budget constraint gives $x_{A1} = \dfrac{m_A}{p_1+p_2} = x_{A2}$.

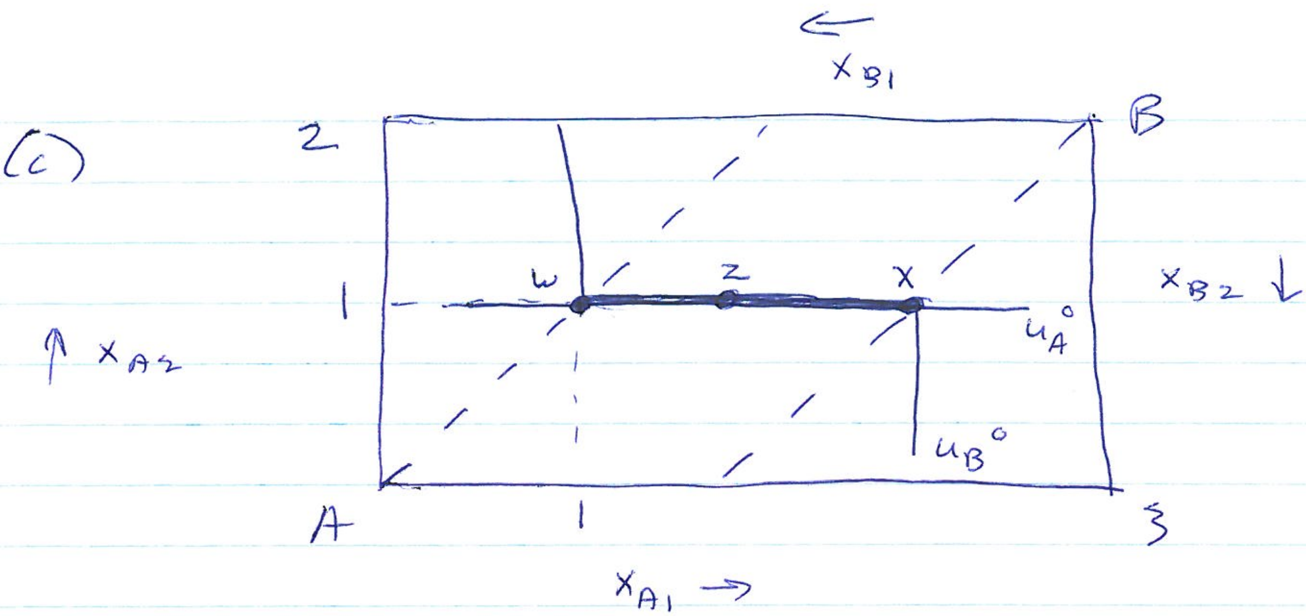
The same reasoning gives $x_{B1} = x_{B2} = \dfrac{m_B}{p_1+p_2}$.
Now use the endowments to get

$$m_A = (1)p_1 + (1)p_2 = p_1 + p_2$$
$$m_B = (2)p_1 + (1)p_2 = 2p_1 + p_2$$

Thus $z_1(p) = x_{A1} + x_{B1} - 3 = 1 + \dfrac{2p_1+p_2}{p_1+p_2} - 3$
and $z_2(p) = x_{A2} + x_{B2} - 2 = 1 + \dfrac{2p_1+p_2}{p_1+p_2} - 2$

Setting $z_1(p) = 0$ gives $2p_1 + p_2 = 2(p_1+p_2) \Rightarrow p_2 = 0$.
Setting $z_2(p) = 0$ gives $2p_1 + p_2 = p_1 + p_2 \Rightarrow p_1 = 0$
The problem is that we <u>assumed</u> $p = (p_1, p_2) > 0$ in deriving the Marshallian demands, but there is no equilibrium at which both prices are positive. Another way to see this is that we can't be at a corner for person A's indiff curve and also at a corner for person B's indiff. curve simultaneously (see graph from part (a)).

(c)



Yes, there is a Walrasian equilibrium associated with the endowment point w from part (b). But for this to work, we need a horizontal budget line through point w. Since A's budget line is

$$p_1 x_{A1} + p_2 x_{A2} = m_A \Rightarrow x_{A2} = \frac{m_A}{p_2} - \frac{p_1 x_{A1}}{p_2}$$

The slope is $-\frac{p_1}{p_2}$ and we need $p_1 = 0$ (set $p_2 > 0$, the exact number doesn't matter)

With a horizontal budget line, A is indifferent toward all points at or to the right of w (these all maximize A's utility). B is indifferent toward all points ~~[illegible]~~ at or to the left of x (again, these all maximize B's utility). Therefore any point along the budget line between w and x (including the endpoints as well as intermediate points like z) is an equilibrium allocation.